Evaluation, Refinement and Fusion of Software-Based Pseudorange Multipath Mitigation Techniques

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The Pseudo-Doppler Observable

1. Start with basic observables:
   \[ \rho_{L1} = d + i + t + m_1 - B + b + \nu + \xi_1 + \varepsilon_1 \]
   \[ \phi_{L1} = d - i + t - B + b + \nu + N_1 \lambda_1 \]
   \[ \rho_{L2} = d + \gamma i + t + m_2 - B + b + \nu + \xi_2 + \varepsilon_2 \]
   \[ \phi_{L2} = d - \gamma i + t - B + b + \nu + N_2 \lambda_2 \]

2. Form the range-phase divergence:
   \[ \Delta_1 = \rho_1 - \phi_1 = 2i + m_1 + \xi_1 - N_1 \lambda_1 + \varepsilon_1 \]

3. Estimate and remove the ionosphere delay:
   \[ i_\phi = \frac{1}{1 - \gamma}(\phi_2 - \phi_1) = i + B_\phi \]
   \[ \mu_1 = \Delta_1 - 2i_\phi = m_1 + B_\mu + \xi_1 + \varepsilon_1 \]

4. Use windowed averages for carrier-phase smoothed pseudoranges:
   \[ \mu_{smoothed} = \Delta_1 - 2i_\phi = m_1 + B_\mu + \bar{\xi}_1 \]
Goal: generate optimal models of pseudorange multipath
First approach: function of a single variable, sidereal time
Ground Track Templates

- Orbit perturbation reduces reliability of sidereal time templates
- Second approach: use ground track longitude:
  
  *Longitude of Sub-Satellite Point, or LSSP*
Using templates to estimate and remove multipath seemingly results in benefits

Multipath Statistics for DOY 340, 2001

- Original observations
- After correction using sidereal templates
However observations cleaned using templates improved marginally for precise positioning.

Positioning Results for DOY 340, 2001

Before

RMS of range residuals: .675 m
Volume of $3\sigma$ error ellipsoid: 16.2 m$^3$

After

RMS of range residuals: .628 m
Volume of $3\sigma$ error ellipsoid: 12.7 m$^3$
Template Model Stability

- Template models are not stable
- LSSP based templates show less instability
- Example of uncertainty growth relative to 3 day templates:

<table>
<thead>
<tr>
<th>All PRNs</th>
<th>Template Type</th>
<th>12 days</th>
<th>24 days</th>
<th>36 days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sidereal</td>
<td>61.7%</td>
<td>82.2%</td>
<td>215.2%</td>
</tr>
<tr>
<td></td>
<td>LSSP</td>
<td>26.4%</td>
<td>32.1%</td>
<td>49.4%</td>
</tr>
</tbody>
</table>

PRN 2

- 3 days
- 36 days

![Graph showing template model stability](image-url)
Template Model Conclusions

- Template models are not stable
- Template models, when used to filter pseudorange observations, provide inconsistent improvement in accuracy.

Why?
- Orbit perturbations reduce correlation of multipath to template parameters: sidereal time, LSSP
- Simple linearized model of ionosphere delay
Kee and Parkinson’s DFM method uses pseudo-Doppler model $\mu$ as function of elevation and azimuth.

Approach: difference $\mu$ where overhead passes cross.

Intersection between two passes

$$
\mu_j = m + B_{\mu_j} + \xi_j + \epsilon_j
$$

$$
\mu_k = m + B_{\mu_k} + \xi_k + \epsilon_k
$$

$$
\Delta \mu_{jk} = \Delta B_{jk} + \Delta \xi_{jk} + \Delta \epsilon_{jk}
$$
Applying the DFM

- Intersection search is computationally expensive
  - Recursive approach developed for this study
- Use least squares approach to estimate the $\Delta B$’s.
  - Forms an estimate of $\Delta B_{j1}$ for each continuous pass $j$
  - One bias remains common to all passes, $B_1$
- After removing $\Delta B$ from passes of $\mu$, remaining multipath can be utilized to
  - Model surfaces: $\rho_{\text{new}} = \rho_{\text{original}} - g(\text{azimuth, elevation})$
  - Model templates: $\rho_{\text{new}} = \rho_{\text{original}} - f(\text{LSSP})$
  - “Directly” remove multipath: $\rho_{\text{new}} = \rho_{\text{original}} - (\mu - \Delta B_j)$
- Surprisingly, direct removal performed the best
  - “Direct” removal does not filter high frequency multipath
  - Effectively this technique **doubles** receiver error
Positioning Results

Observations cleaned using DFM to estimate biases performed significantly better in precise positioning

Positioning Results for DOY 340, 2001

Before

<table>
<thead>
<tr>
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<th>East (m)</th>
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<tbody>
<tr>
<td>-2</td>
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<tr>
<td>-1.5</td>
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RMS of range residuals: 0.675 m
Volume of $3\sigma$ error ellipsoid: 16.2 m$^3$

After

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RMS of range residuals: 0.464 m
Volume of $3\sigma$ error ellipsoid: 3.2 m$^3$
Conclusions

♦ LSSP templates perform better than sidereal
♦ Template models are unstable
♦ Perhaps improved ionosphere model would improve the performance of templates and surfaces
♦ Reduced $\sigma_\mu$ necessary but not sufficient for mitigation
♦ Using DFM for bias estimates, without generating a surface model, results in best mitigation strategy
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